How to Make a Decision: The Analytic Hierarchy Process

THOMAS L. SAATY

322 Mervis Hall
University of Pittsburgh
Pittsburgh, Pennsylvania 15260

People make three general types of judgments to express importance, preference, or likelihood and use them to choose the best among alternatives in the presence of environmental, social, political, and other influences. They base these judgments on knowledge in memory or from analyzing benefits, costs, and risks. From past knowledge, we sometimes can develop standards of excellence and poorness and use them to rate the alternatives one at a time. This is useful in such repetitive situations as student admissions and salary raises that must conform with established norms. Without norms one compares alternatives instead of rating them. Comparisons must fall in an admissible range of consistency. The analytic hierarchy process (AHP) includes both the rating and comparison methods. Rationality requires developing a reliable hierarchic structure or feedback network that includes criteria of various types of influence, stakeholders, and decision alternatives to determine the best choice.

Policy makers at all levels of decision making in organizations use multiple criteria to analyze their complex problems. Multicriteria thinking is used formally to facilitate their decision making. Through trade-offs it clarifies the advantages and disadvantages of policy options under circumstances of risk and uncertainty. It is

also a tool vital to forming corporate strategies needed for effective competition.

Nearly all of us, in one way or another, have been brought up to believe that clear-headed logical thinking is our only sure way to face and solve problems. We also believe that our feelings and our judgments must be subjected to the acid test of deductive thinking. But experience suggests that deductive thinking is not natural. Indeed, we have to practice, and for a long time, before we can do it well. Since complex problems usually have many related factors, traditional logical thinking leads to sequences of ideas that are so tangled that their interconnections are not readily discerned.

The lack of a coherent procedure to make decisions is especially troublesome

---

We have been brought up to believe that clear-headed logical thinking is our only sure way to solve problems.

---

when our intuition alone cannot help us to determine which of several options is the most desirable, or the least objectionable, and neither logic nor intuition are of help. Therefore, we need a way to determine which objective outweighs another, both in the near and long terms. Since we are concerned with real-life problems we must recognize the necessity for trade-offs to best serve the common interest. Therefore, this process should also allow for consensus building and compromise.

Individual knowledge and experience are inadequate in making decisions concerning the welfare and quality of life for a group. Participation and debate are needed both among individuals and between the groups affected. Here two aspects of group decision making have to be considered. The first is a rather minor complication, namely, the discussion and exchange within the group to reach some kind of consensus on the given problem. The second is of much greater difficulty. The holistic nature of the given problem necessitates that it be divided into smaller subject-matter areas within which different groups of experts determine how each area affects the total problem. A large and complex problem can rarely be decomposed simply into a number of smaller problems whose solutions can be combined into an overall answer. If this process is successful, one can then reconstruct the initial question and review the proposed solutions. A last and often crucial disadvantage of many traditional decision-making methods is that they require specialized expertise to design the appropriate structure and then to embed the decision-making process in it.

A decision-making approach should have the following characteristics:
—Be simple in construct,
—Be adaptable to both groups and individuals,
—Be natural to our intuition and general thinking,
—Encourage compromise and consensus building, and
—Not require inordinate specialization to master and communicate [Saaty 1982].
In addition, the details of the processes leading up to the decision-making process should be easy to review.

At the core of the problems that our method addresses is the need to assess the

benefits, the costs, and the risks of the proposed solutions. We must answer such questions as the following: Which consequences weigh more heavily than others? Which aims are more important than others? What is likely to take place? What should we plan for and how do we bring it about? These and other questions demand a multicriteria logic. It has been demonstrated over and over by practitioners who use the theory discussed in this paper that multicriteria logic gives different and often better answers to these questions than ordinary logic and does it efficiently.

To make a decision one needs various kinds of knowledge, information, and technical data. These concern

—Details about the problem for which a decision is needed,

—The people or actors involved,

—Their objectives and policies,

—The influences affecting the outcomes, and

—The time horizons, scenarios, and constraints.

The set of potential outcomes or alternatives from which to choose are the essence of decision making. In laying out the framework for making a decision, one needs to sort the elements into groupings or clusters that have similar influences or effects. One must also arrange them in some rational order to trace the outcome of these influences. Briefly, we see decision making as a process that involves the following steps:

(1) Structure a problem with a model that shows the problem's key elements and their relationships.

(2) Elicit judgments that reflect knowledge, feelings, or emotions.

(3) Represent those judgments with meaningful numbers.

(4) Use these numbers to calculate the priorities of the elements of the hierarchy.

(5) Synthesize these results to determine an overall outcome.

(6) Analyze sensitivity to changes in judgment [Saaty 1977].

The decision-making process described in this paper meets these criteria. I call it the analytic hierarchy process (AHP). The

Deductive thinking is not natural.

AHP is about breaking a problem down and then aggregating the solutions of all the subproblems into a conclusion. It facilitates decision making by organizing perceptions, feelings, judgments, and memories into a framework that exhibits the forces that influence a decision. In the simple and most common case, the forces are arranged from the more general and less controllable to the more specific and controllable. The AHP is based on the innate human ability to make sound judgments about small problems. It has been applied in a variety of decisions and planning projects in nearly 20 countries.

Here *rationality* is

—Focusing on the goal of solving the problem;

—Knowing enough about a problem to develop a complete structure of relations and influences;

—Having enough knowledge and experience and access to the knowledge and experience of others to assess the prior-

ity of influence and dominance (importance, preference, or likelihood to the goal as appropriate) among the relations in the structure;

—Allowing for differences in opinion with an ability to develop a best compromise.

How to Structure a Hierarchy

Perhaps the most creative part of decision making that has a significant effect on the outcome is modeling the problem. In the AHP, a problem is structured as a hierarchy. This is then followed by a process of prioritization, which we describe in detail later. Prioritization involves eliciting judgments in response to questions about the dominance of one element over another when compared with respect to a property. The basic principle to follow in creating this structure is always to see if one can answer the following question: Can I compare the elements on a lower level using some or all of the elements on the next higher level as criteria or attributes of the lower level elements?

A useful way to proceed in structuring a decision is to come down from the goal as far as one can by decomposing it into the most general and most easily controlled factors. One can then go up from the alternatives beginning with the simplest subcriteria that they must satisfy and aggregating the subcriteria into generic higher level criteria until the levels of the two processes are linked in such a way as to make comparison possible.

Here are some suggestions for an elaborate design of a hierarchy: (1) Identify the overall goal. What are you trying to accomplish? What is the main question? (2) Identify the subgoals of the overall goal. If relevant, identify time horizons that affect

the decision. (3) Identify criteria that must be satisfied to fulfill the subgoals of the overall goal. (4) Identify subcriteria under each criterion. Note that criteria or subcriteria may be specified in terms of ranges of values of parameters or in terms of verbal intensities such as high, medium, low. (5) Identify the actors involved. (6) Identify the actors' goals. (7) Identify the actors' policies. (8) Identify options or outcomes. (9) For yes-no decisions, take the most preferred outcome and compare the benefits and costs of making the decision with those of not making it. (10) Do a benefit/cost analysis using marginal values. Because we are dealing with dominance hierarchies, ask which alternative yields the greatest benefit; for costs, which alternative costs the most, and for risks, which alternative is more risky.

The Hospice Problem

Westmoreland County Hospital in Western Pennsylvania, like hospitals in many other counties around the nation, has been concerned with the costs of the facilities and manpower involved in taking care of terminally ill patients. Normally these patients do not need as much medical attention as do other patients. Those who best utilize the limited resources in a hospital are patients who require the medical attention of its specialists and advanced technology equipment—whose utilization depends on the demand of patients admitted into the hospital. The terminally ill need medical attention only episodically. Most of the time such patients need psychological support. Such support is best given by the patient's family, whose members are able to supply the love and care the patients most need. For the mental health of

the patient, home therapy is a benefit. From the medical standpoint, especially during a crisis, the hospital provides a greater benefit. Most patients need the help of medical professionals only during a crisis. Some will also need equipment and surgery. The planning association of the hospital wanted to develop alternatives and to choose the best one considering various criteria from the standpoint of the patient, the hospital, the community, and society at large. In this problem, we need to consider the costs and benefits of the decision. Cost includes economic costs and all sorts of intangibles, such as inconvenience and pain. Such disbenefits are not directly related to benefits as their mathematical inverses, because patients infinitely prefer the benefits of good health to these intangible disbenefits. To study the problem, one needs to deal with benefits and with costs separately.

Approaching the Problem

I met with representatives of the planning association for several hours to decide on the best alternative. To make a decision by considering benefits and costs, one must first answer the question: In this problem, do the benefits justify the costs? If they do, then either the benefits are so much more important than the costs that the decision is based simply on benefits, or the two are so close in value that both the benefits and the costs should be considered. Then we use two hierarchies for the purpose and make the choice by forming ratios of the priorities of the alternatives (benefits b_i/costs c_i) from them. One asks which is most beneficial in the benefits hierarchy (Figure 1) and which is most costly in the costs hierarchy (Figure 2). If the

benefits do not justify the costs, the costs alone determine the best alternative—that which is the least costly. In this example, we decided that both benefits and costs had to be considered in separate hierarchies. In a risk problem, a third hierarchy is used to determine the most desired alternative with respect to all three: benefits, costs, and risks. In this problem, we assumed risk to be the same for all contingencies. Whereas for most decisions one uses only a single hierarchy, we constructed two hierarchies for the hospice problem, one for benefits or gains (which model of hospice care yields the greater benefit) and one for costs or pains (which model costs more).

The planning association thought the concepts of benefits and costs were too general to enable it to make a decision. Thus, the planners and I further subdivided each (benefits and costs) into detailed subcriteria to enable the group to develop alternatives and to evaluate the finer distinctions the members perceived between the three alternatives. The alternatives were to care for terminally ill patients at the hospital, at home, or partly at the hospital and partly at home.

For each of the two hierarchies, benefits and costs, the goal clearly had to be choosing the best hospice. We placed this goal at the top of each hierarchy. Then the group discussed and identified overall criteria for each hierarchy; these criteria need not be the same for the benefits as for the costs.

The two hierarchies are fairly clear and straightforward in their description. They descend from the more general criteria in the second level to secondary subcriteria in the third level and then to tertiary subcri-
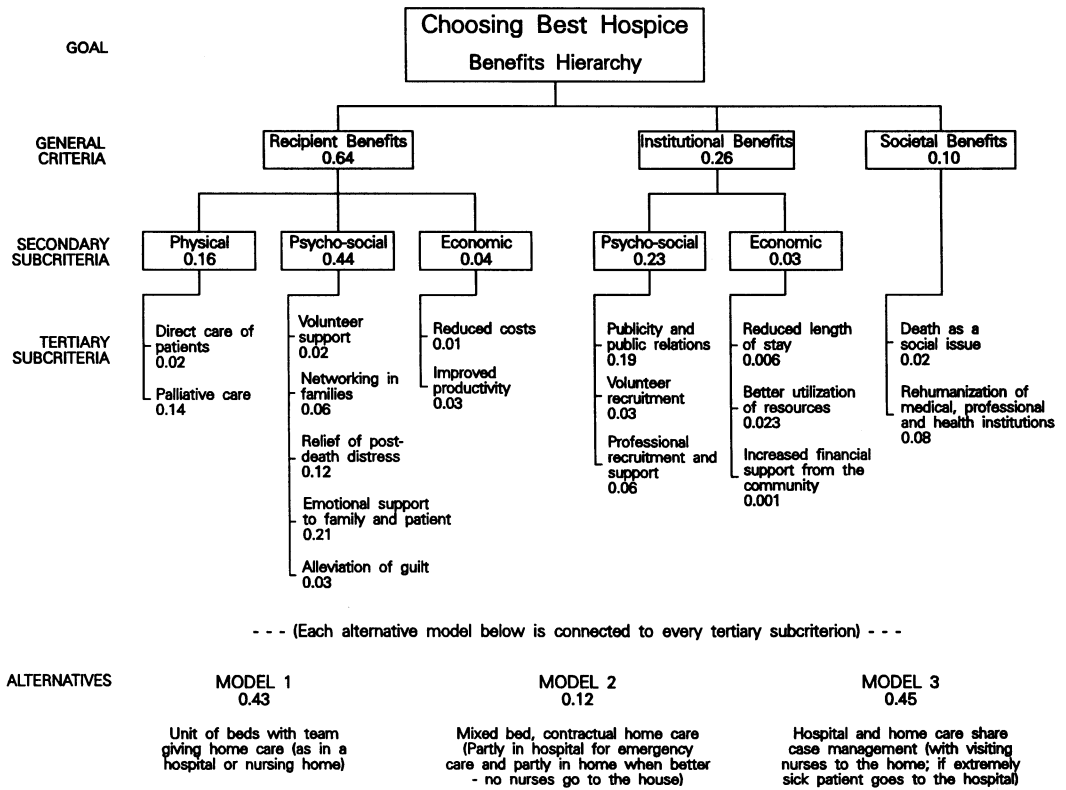
Figure 1: To choose the best hospice plan, one constructs a hierarchy modeling the benefits to the patient, to the institution, and to society. This is the benefits hierarchy of two separate hierarchies.

teria in the fourth level on to the alternatives at the bottom or fifth level.

At the general criteria level, each of the hierarchies, benefits or costs, involved three major interests. The decision should benefit the recipient, the institution, and society as a whole, and their relative importance is the prime determinant as to which outcome is more likely to be preferred. We located these three elements on the second level of the benefits hierarchy. As the decision would benefit each party differently and the importance of the benefits to each recipient affects the outcome, the group thought that it was important to specify the types of benefit for the recipi-

ent and the institution. Recipients want physical, psycho-social and economic benefits, while the institution wants only psychosocial and economic benefits. We located these benefits in the third level of the hierarchy. Each of these in turn needed further decomposition into specific items in terms of which the decision alternatives could be evaluated. For example, while the recipient measures economic benefits in terms of reduced costs and improved productivity, the institution needed the more specific measurements of reduced length of stay, better utilization of resources, and increased financial support from the community. There was no reason to decompose
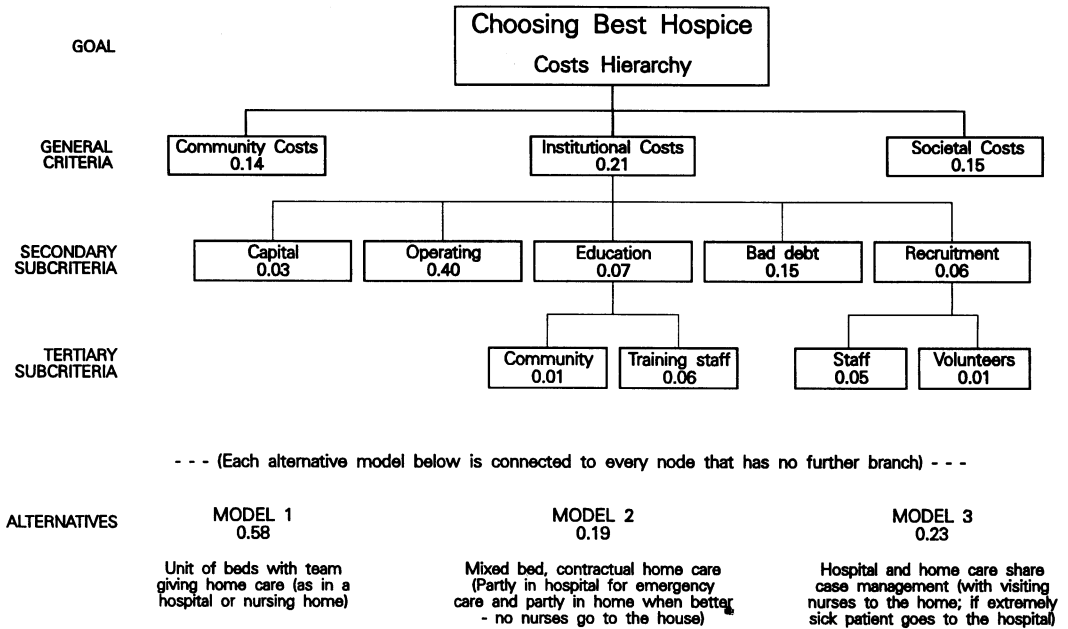
Figure 2: To choose the best hospice plan, one constructs a hierarchy modeling the community, institutional, and societal costs. This is the costs hierarchy of two separate hierarchies.

the societal benefits into a third level sub-criteria, hence societal benefits connects directly to the fourth level. The group considered three models for the decision alternatives, and located them on the bottom or fifth level of the hierarchy: In Model 1, the hospital provided full care to the patients; In Model 2, the family cares for the patient at home, and the hospital provides only emergency treatment (no nurses go to the house); and in Model 3, the hospital and the home share patient care (with visiting nurses going to the home).

In the costs hierarchy there were also three major interests in the second level that would incur costs or pains: community, institution, and society. In this decision the costs incurred by the patient were not included as a separate factor. Patient and family could be thought of as part of the community. We thought decomposi-

tion was necessary only for institutional costs. We included five such costs in the third level: capital costs, operating costs, education costs, bad debt costs, and recruitment costs. Educational costs apply to educating the community and training the staff. Recruitment costs apply to staff and volunteers. Since both the costs hierarchy and the benefits hierarchy concern the same decision, they both have the same alternatives in their bottom levels, even though the costs hierarchy has fewer levels.

Judgments and Comparisons

A judgment or comparison is the numerical representation of a relationship between two elements that share a common parent. The set of all such judgments can be represented in a square matrix in which the set of elements is compared with itself. Each judgment represents the dominance

of an element in the column on the left over an element in the row on top. It reflects the answers to two questions: Which of the two elements is more important with respect to a higher level criterion, and how strongly, using the 1–9 scale shown in Table 1 for the element on the left over the element at the top of the matrix? If the element on the left is less important than that on the top of the matrix, we enter the reciprocal value in the corresponding position in the matrix. It is important to note that the lesser element is always used as the unit and the greater one is estimated as a multiple of that unit. From all the paired comparisons we calculate the priorities and exhibit them on the right of the matrix. For a set of n elements in a matrix one needs $n(n-1)/2$ comparisons because there are n 1's on the diagonal for comparing ele-

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective.
3	Moderate importance	Experience and judgment slightly favor one activity over another.
5	Strong importance	Experience and judgment strongly favor one activity over another.
7	Very strong or demonstrated importance	An activity is favored very strongly over another, its dominance demonstrated in practice.
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
2, 4, 6, 8	For compromise between the above values	Sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it.
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	A comparison mandated by choosing the smaller element as the unit to estimate the larger one as a multiple of that unit.
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix.
1.1–1.9	For tied activities	When elements are close and nearly indistinguishable; moderate is 1.3 and extreme is 1.9.

Table 1: The fundamental scale is a scale of absolute numbers used to assign numerical values to judgments made by comparing two elements with the smaller element used as the unit and the larger one assigned a value from this scale as a multiple of that unit.

ments with themselves and of the remaining judgments, half are reciprocals. Thus we have $(n^2 - n)/2$ judgments. In some problems one may elicit only the minimum of $n - 1$ judgments.

As usual with the AHP, in both the cost and the benefits models, we compared the criteria and subcriteria according to their relative importance with respect to the parent element in the adjacent upper level. For example, in the first matrix of comparisons of the three benefits criteria with respect to the goal of choosing the best hospice alternative, recipient benefits are moderately more important than institutional benefits and are assigned the absolute number 3 in the (1, 2) or first-row, second-column position. Three signifies three times more. The reciprocal value is automatically entered in the (2, 1) position, where institutional benefits on the left are compared with recipient benefits at the top. Similarly a 5, corresponding to strong dominance or importance, is assigned to recipient benefits over social benefits in the (1, 3) position, and a 3, corresponding to moderate dominance, is assigned to institutional benefits over social benefits in the (2, 3) position with corresponding reciprocals in the transpose positions of the matrix.

Judgments in a matrix may not be consistent. In eliciting judgments, one makes redundant comparisons to improve the validity of the answer, given that respondents may be uncertain or may make poor judgments in comparing some of the elements. Redundancy gives rise to multiple comparisons of an element with other elements and hence to numerical inconsistencies. For example, where we compare recipient benefits with institutional benefits and with societal benefits, we have the respective judgments 3 and 5. Now if $x = 3y$ and $x = 5z$ then $3y = 5z$ or $y = 5/3z$. If the judges were consistent, institutional benefits would be assigned the value $5/3$ instead of the 3 given in the matrix. Thus the judgments are inconsistent. In fact, we are not sure which judgments are more accurate and which are the cause of the inconsistency. Inconsistency is inherent in the judgment process. Inconsistency may be considered a tolerable error in measurement only when it is of a lower order of magnitude (10 percent) than the actual measurement itself; otherwise the inconsistency would bias the result by a sizable error comparable to or exceeding the actual measurement itself.

When the judgments are inconsistent, the decision maker may not know where the greatest inconsistency is. The AHP can

Choosing Best Hospice	Recipient Benefits	Institutional Benefits	Social Benefits	Priorities
Recipient Benefits	1	3	5	.64
Institutional Benefits	1/3	1	3	.26
Societal Benefits	1/5	1/3	1	.11

C.R. = .033

Table 2: The entries in this matrix respond to the question, Which criterion is more important with respect to choosing the best hospice alternative and how strongly?

show one by one in sequential order which judgments are the most inconsistent, and also suggests the value that best improves consistency. However, this recommendation may not necessarily lead to a more accurate set of priorities that correspond to some underlying preference of the decision

A decision-making approach should be natural to our intuition and general thinking.

makers. Greater consistency does not imply greater accuracy and one should go about improving consistency (if one can be given the available knowledge) by making slight changes compatible with one's understanding. If one cannot reach an acceptable level of consistency, one should gather more information or reexamine the framework of the hierarchy.

Under each matrix I have indicated a consistency ratio (CR) comparing the inconsistency of the set of judgments in that matrix with what it would be if the judgments and the corresponding reciprocals were taken at random from the scale. For a 3-by-3 matrix this ratio should be about five percent, for a 4-by-4 about eight percent, and for larger matrices, about 10 percent.

Priorities are numerical ranks measured on a ratio scale. A ratio scale is a set of positive numbers whose ratios remain the same if all the numbers are multiplied by an arbitrary positive number. An example is the scale used to measure weight. The ratio of these weights is the same in pounds and in kilograms. Here one scale is just a constant multiple of the other. The

object of evaluation is to elicit judgments concerning relative importance of the elements of the hierarchy to create scales of priority of influence.

Because the benefits priorities of the alternatives at the bottom level belong to a ratio scale and their costs priorities also belong to a ratio scale, and since the product or quotient (but not the sum or the difference) of two ratio scales is also a ratio scale, to derive the answer we divide the benefits priority of each alternative by its costs priority. We then choose the alternative with the largest of these ratios. It is also possible to allocate a resource proportionately among the alternatives.

I will explain how priorities are developed from judgments and how they are synthesized down the hierarchy by a process of weighting and adding to go from local priorities derived from judgments with respect to a single criterion to global priorities derived from multiplication by the priority of the criterion and overall priorities derived by adding the global priorities of the same element. The local priorities are listed on the right of each matrix. If the judgments are perfectly consistent, and hence $CR = 0$, we obtain the local priorities by adding the values in each row and dividing by the sum of all the judgments, or simply by normalizing the judgments in any column, by dividing each entry by the sum of the entries in that column. If the judgments are inconsistent but have a tolerable level of inconsistency, we obtain the priorities by raising the matrix to large powers, which is known to take into consideration all intransitivities between the elements, such as those I showed above between x, y, and z [Saaty

1994]. Again, we obtain the priorities from this matrix by adding the judgment values in each row and dividing by the sum of all the judgments. To summarize, the global priorities at the level immediately under the goal are equal to the local priorities because the priority of the goal is equal to one. The global priorities at the next level are obtained by weighting the local priorities of this level by the global priority at the level immediately above and so on. The overall priorities of the alternatives are obtained by weighting the local priorities by the global priorities of all the parent criteria or subcriteria in terms of which they are compared and then adding. (If an element in a set is not comparable with the others on some property and should be left out, the local priorities can be augmented by adding a zero in the appropriate position.)

The process is repeated in all the matrices by asking the appropriate dominance or importance question. For example, for the matrix comparing the subcriteria of the parent criterion institutional benefits (Table 3), psychosocial benefits are regarded as very strongly more important than economic benefits, and 7 is entered in the (1, 2) position and 1/7 in the (2, 1) position.

Institutional Benefits	Psychosocial	Economic	Priorities
Psychosocial	1	7	.875
Economics	1/7	1	.125

C.R. = .000

Table 3: The entries in this matrix respond to the question, Which subcriterion yields the greater benefit with respect to institutional benefits and how strongly?

In comparing the three models for patient care, we asked members of the planning association which model they preferred with respect to each of the covering or parent secondary criterion in level 3 or with respect to the tertiary criteria in level 4. For example, for the subcriterion direct care (located on the left-most branch in the benefits hierarchy), we obtained a matrix of paired comparisons (Table 4) in which Model 1 is preferred over Models 2 and 3 by 5 and 3 respectively, and Model 3 is preferred by 3 over Model 2. The group first made all the comparisons using semantic terms for the fundamental scale and then translated them to the corresponding numbers.

For the costs hierarchy, I again illustrate with three matrices. First the group compared the three major cost criteria and provided judgments in response to the question: which criterion is a more important determinant of the cost of a hospice model? Table 5 shows the judgments obtained.

The group then compared the subcriteria under institutional costs and obtained the importance matrix shown in Table 6.

Finally we compared the three models to find out which incurs the highest cost for each criterion or subcriterion. Table 7 shows the results of comparing them with respect to the costs of recruiting staff. As shown in Table 8, we divided the benefits priorities by the costs priorities for each alternative to obtain the best alternative, model 3, the one with the largest value for the ratio.

Table 8 shows two ways or modes of synthesizing the local priorities of the alternatives using the global priorities of

Direct Care of Patient	Model I	Model II	Model III	Priorities
Model I— Unit/Team	1	5	3	0.64
Model II— Mixed/ Home Care	1/5	1	1/3	0.10
Model III— Case Management	1/3	3	1	0.26

C.R. = .033

Table 4: The entries in this matrix respond to the question, Which model yields the greater benefit with respect to direct care of the patient and how strongly?

their parent criteria: The distributive mode and the ideal mode. In the distributive mode, the weights of the alternatives sum to one. It is used when there is dependence among the alternatives and a unit priority is distributed among them. The ideal mode is used to obtain the single best alternative regardless of what other alternatives there are. In the ideal mode, the local priorities of the alternatives are divided by the largest value among them. This is done for each criterion; for each criterion one alternative becomes an ideal with value one. In both modes, the local priorities are weighted by the global priorities of the parent criteria and synthesized and the

benefit-to-cost ratios formed. In this case, both modes lead to the same outcome for hospice, which is model 3. As we shall see below, we need both modes to deal with the effect of adding (or deleting) alternatives on an already ranked set.

Model 3 has the largest ratio scale values of benefits to costs in both the distributive and ideal modes, and the hospital selected it for treating terminal patients. This need not always be the case. In this case, there is dependence of the personnel resources allocated to the three models because some of these resources would be shifted based on the decision. Therefore the distributive mode is the appropriate method of synthe-

Choosing Best Hospice (Costs)	Community	Institutional	Societal	Priorities
Community Costs	1	1/5	1	0.14
Institutional Costs	5	1	5	0.71
Societal Costs	1	1/5	1	0.14

C.R. = .000

Table 5: The entries in this matrix respond to the question, Which criterion is a greater determinant of cost with respect to the care method and how strongly?

Institutional Costs	Capital	Operating	Education	Bad Debt	Recruitment	Priorities
Capital	1	1/7	1/4	1/7	1	0.05
Operating	7	1	9	4	5	0.57
Education	4	1/9	1	1/2	1	0.10
Bad Debt	7	1/4	2	1	3	0.21
Recruitment	1	1/5	1	1/3	1	0.07

C.R. = .08

Table 6: The entries in this matrix respond to the question, Which criterion incurs greater institutional costs and how strongly?

sis. If the alternatives were sufficiently distinct with no dependence in their definition, the ideal mode would be the way to synthesize.

I also performed marginal analysis to determine where the hospital should allocate additional resources for the greatest marginal return. To perform marginal analysis, I first ordered the alternatives by increasing cost priorities and then formed the benefit-to-cost ratios corresponding to the smallest cost, followed by the ratios of the differences of successive benefits to costs. If this difference in benefits is negative, the new alternative is dropped from consideration and the process continued. The alternative with the largest marginal ratio is then chosen. For the costs and corresponding benefits from the synthesis rows in Table 8 I obtained:

Costs: 0.20   0.21   0.59
Benefits: 0.12   0.45   0.43

$$\text{Marginal Ratios:} \quad \frac{0.12}{0.20} = 0.60$$

$$\frac{0.45 - 0.12}{0.21 - 0.20} = 33$$

$$\frac{0.43 - 0.45}{0.59 - 0.21} = -0.05$$

The third alternative is not a contender for resources because its marginal return is negative. The second alternative is best. In fact, in addition to adopting the third model, the hospital management chose the

Institutional Costs for Recruiting Staff	Model I	Model II	Model III	Priorities
Model I—Unit/ Team	1	4	4	.66
Model II—Mixed/ Home Care	1/4	1	1	.17
Model III—Case Management	1/4	1	1	.17

C.R. = .000

Table 7: The entries in this matrix respond to the question, Which model incurs greater cost with respect to institutional costs for recruiting staff and how strongly?

Benefits	Priorities	Distributive Mode			Ideal Mode		
		Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Direct Care of Patient	0.02	0.64	0.10	0.26	1.000	0.156	0.406
Palliative Care	0.14	0.64	0.10	0.26	1.000	0.156	0.406
Volunteer Support	0.02	0.09	0.17	0.74	0.122	0.230	1.000
Networking in Families	0.06	0.46	0.22	0.32	1.000	0.478	0.696
Relief of Post Death Stress	0.12	0.30	0.08	0.62	0.484	0.129	1.000
Emotional Support of Family and Patient	0.21	0.30	0.08	0.62	0.484	0.129	1.000
Alleviation of Guilt	0.03	0.30	0.08	0.62	0.484	0.129	1.000
Reduced Economic Costs for Patient	0.01	0.12	0.65	0.23	0.185	1.000	0.354
Improved Productivity	0.03	0.12	0.27	0.61	0.197	0.443	1.000
Publicity and Public Relations	0.19	0.63	0.08	0.29	1.000	0.127	0.460
Volunteer Recruitment	0.03	0.64	0.10	0.26	1.000	0.156	0.406
Professional Recruitment and Support	0.06	0.65	0.23	0.12	1.000	0.354	0.185
Reduced Length of Stay	0.006	0.26	0.10	0.64	0.406	0.406	1.000
Better Utilization of Resources	0.023	0.09	0.22	0.69	0.130	0.130	1.000
Increased Monetary Support	0.001	0.73	0.08	0.19	1.000	1.000	0.260
Death as a Social Issue	0.02	0.20	0.20	0.60	0.333	0.333	1.000
Rehumanization of Institutions	0.08	0.24	0.14	0.62	0.387	0.226	1.000
Synthesis		0.428	0.121	0.451	0.424	0.123	0.453
Costs							
Community Costs	0.14	0.33	0.33	0.33	1.000	1.000	1.000
Institutional Capital Costs	0.03	0.76	0.09	0.15	1.000	0.118	0.197
Institutional Operating Costs	0.40	0.73	0.08	0.19	1.000	0.110	0.260
Institutional Costs for Educating the Community	0.01	0.65	0.24	0.11	1.000	0.369	0.169
Institutional Costs for Training Staff	0.06	0.56	0.32	0.12	1.000	0.571	0.214
Institutional Bad Debt	0.15	0.60	0.20	0.20	1.000	0.333	0.333
Institutional Costs of Recruiting Staff	0.05	0.66	0.17	0.17	1.000	0.258	0.258
Institutional Costs of Recruiting Volunteers	0.01	0.60	0.20	0.20	1.000	0.333	0.333
Societal Costs	0.15	0.33	0.33	0.33	1.000	1.000	1.000
Synthesis		0.583	0.192	0.224	0.523	0.229	0.249
Benefit/Cost Ratio		0.734	0.630	2.013	0.811	0.537	1.819

Table 8: The benefit/cost ratios of the three models given in the bottom row of the table are obtained for both the distributive and ideal modes. Here one multiplies each of the six columns of priorities of a model by the column of criteria weights on the left and adds to obtain the synthesis of overall priorities, once for the benefits (top half of table) and once for the costs (bottom half of table) and forms the ratios of corresponding synthesis numbers to arrive at the benefit/cost ratio (bottom row of table).

second model of hospice care for further development.

Absolute Measurement

Cognitive psychologists have recognized for some time that people are able to make two kinds of comparisons—absolute and relative. In absolute comparisons, people compare alternatives with a standard in their memory that they have developed through experience. In relative comparisons, they compared alternatives in pairs according to a common attribute, as we did throughout the hospice example.

People use absolute measurement (sometimes also called rating) to rank independent alternatives one at a time in terms of rating intensities for each of the criteria. An intensity is a range of variation of a criterion that enables one to distinguish the quality of an alternative for that criterion. An intensity may be expressed as a numerical range of values if the criterion is measurable or in qualitative terms. For example, if ranking students is the objective and one of the criteria on which they are to be ranked is performance in mathematics, the mathematics ratings might be: excellent, good, average, below average, poor; or, using the usual school terminology, A, B, C, D, and F. Relative comparisons are first used to set priorities on the ratings themselves. If desired, one can fit a continuous curve through the derived intensities. This concept may go against our socialization. However, it is perfectly reasonable to ask how much an A is preferred to a B or to a C. The judgment of how much an A is preferred to a B might be different under different criteria. Perhaps for mathematics an A is very strongly preferred to a B, while for physical education an A is only

moderately preferred to a B. So the end result might be that the ratings are scaled differently. For example one could have the following scale values for the ratings:

	MATH	PHYSICAL EDUCATION
A	0.50	0.30
B	0.30	0.30
C	0.15	0.20
D	0.04	0.10
E	0.01	0.10

The alternatives are then rated or ticked off one at a time on the intensities.

I will illustrate absolute measurement with an example. A firm evaluates its employees for raises. The criteria are dependability, education, experience, and quality. Each criterion is subdivided into intensities, standards, or subcriteria (Figure 3). The managers set priorities for the criteria by comparing them in pairs. They then pairwise compare the intensities according to priority with respect to their parent criterion (as in Table 9) or with respect to a subcriterion if they are using a deeper hierarchy. The priorities of the intensities are divided by the largest intensity for each criterion (second column of priorities in Figure 3). Table 9 shows a paired comparison matrix of intensities with respect to dependability. The managers answer the question, Which intensity is more important and by how much with respect to dependability? The answer will depend on the kind of job. "Outstanding" is much more preferred over "above average" for a soldier guarding a nuclear missile sight than for a waiter in a restaurant. Comparison of intensities requires expert judgment

GOAL

| Dependability .4347 | Education .2774 | Experience .1755 | Quality .1123 |

- Outstanding (0.182) 1.000
- Above Average (0.114) 0.626
- Average (0.070) 0.385
- Below Average (0.042) 0.231
- Unsatisfactory (0.027) 0.148

- Doctorate (0.144) 1.000
- Masters (0.071) 0.493
- Bachelor (0.041) 0.285
- H.S. (0.014) 0.097
- Uneducated (0.007) 0.049

- Exceptional (0.086) 1.000
- A Lot (0.050) 0.580
- Average (0.023) 0.267
- A Little (0.010) 0.116
- None (0.006) 0.070

- Outstanding (0.056) 1.000
- Above Average (0.029) 0.518
- Average (0.018) 0.321
- Below Average (0.006) 0.107
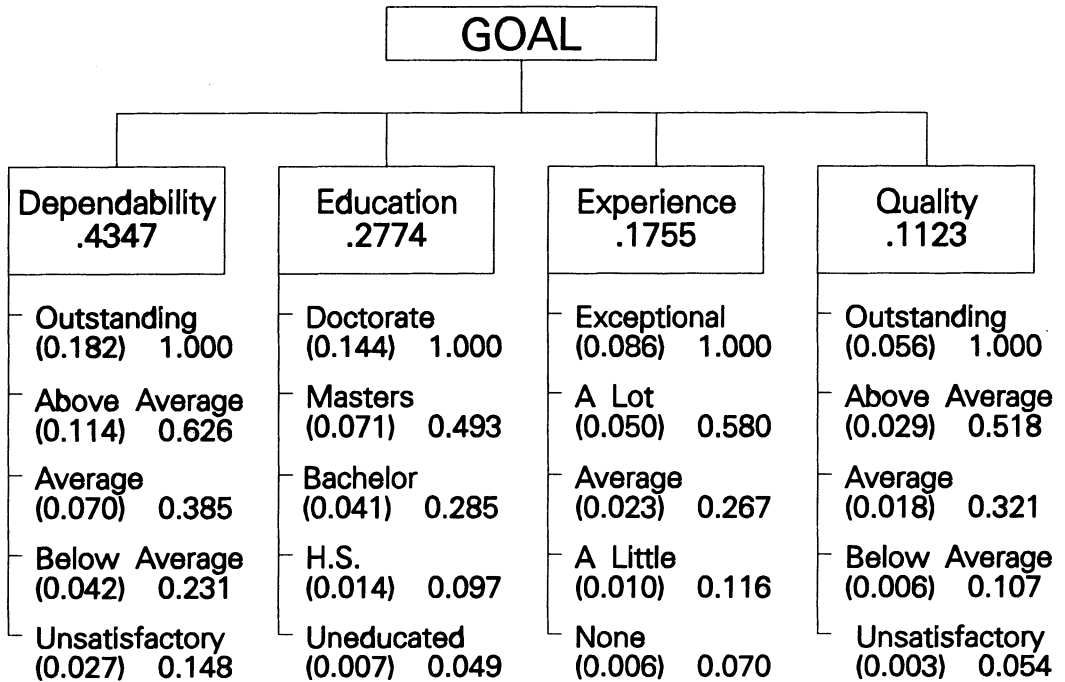- Unsatisfactory (0.003) 0.054

Figure 3: An evaluation hierarchy can be used to rate employees.

in each problem and for each criterion. Finally, the managers rate each individual (Table 10) by assigning the intensity rating that applies to him or her under each criterion. The scores of these intensities are each weighted by the priority of its criterion and summed to derive a total ratio scale score for the individual (shown on the right of Table 10). These numbers belong to a ratio scale, and the managers can give salary increases precisely in proportion to the ratios of these numbers. Adams gets the highest score and Kesselman the lowest. This approach can be used whenever it is possible to set priorities for intensities of criteria; people can usually do this when they have sufficient experience with a given operation. This normative mode

	Outstanding	Above Average	Average	Below Average	Unsatisfactory	Priorities
Outstanding	1.0	2.0	3.0	4.0	5.0	0.419
Above Average	1/2	1.0	2.0	3.0	4.0	0.263
Average	1/3	1/2	1.0	2.0	3.0	0.160
Below Average	1/4	1/3	1/2	1.0	2.0	0.097
Unsatisfactory	1/5	1/4	1/3	1/2	1.0	0.062

C.R. = 0.015

Table 9: Ranking intensities: Which intensity is preferred most with respect to dependability and how strongly?

	Dependability 0.4347	Education 0.2774	Experience 0.1775	Quality 0.1123	Total
1. Adams, V.	Outstanding	Bachelor	A Little	Outstanding	0.646
2. Becker, L.	Average	Bachelor	A Little	Outstanding	0.379
3. Hayat, F.	Average	Masters	A Lot	Below Average	0.418
4. Kesselman, S.	Above Average	H.S.	None	Above Average	0.369
5. O'Shea, K.	Average	Doctorate	A Lot	Above Average	0.605
6. Peters, T.	Average	Doctorate	A Lot	Average	0.583
7. Tobias, K.	Above Average	Bachelor	Average	Above Average	0.456

Table 10: Ranking alternatives. The priorities of the intensities for each criterion are divided by the largest one and multiplied by the priority of the criterion. Each alternative is rated on each criterion by assigning the appropriate intensity. The weighted intensities are added to yield the total on the right.

requires that alternatives be rated one by one without regard to how many there may be and how high or low any of them rates on prior standards. Some corporations have insisted that they no longer trust the normative standards of their experts and that they prefer to make paired comparisons of their alternatives. Still, when there is wide agreement on standards, the absolute mode saves time in rating a large number of alternatives.

Homogeneity and Clustering

Think of the following situation: we need to determine the relative size of a blueberry and a watermelon. Here, we need a range greater than 1–9. Human beings have difficulty establishing appropriate relationships when the ratios get beyond 9. To resolve this human difficulty, we can use a method in which we cluster different elements so we can rate them within a cluster and then rate them across the clusters. We need to add other fruits to make the comparison possible and then form groups of comparable fruits. In the first group we include the blueberry, a grape, and a plum. In the second group we include the same plum, an apple, and a

grapefruit. In the third group we include the same grapefruit, a melon, and the watermelon. The AHP requires reciprocal comparisons of homogeneous elements whose ratios do not differ by much on a property, hence the absolute scale 1–9; when the ratios are larger, one must cluster the elements in different groups and use a common element (pivot) that is the largest in one cluster and the smallest element in the next cluster of the next higher order of magnitude. The weights of the elements in the second group are divided by the priority of the pivot in that group and then multiplied by the priority of the same pivot element (whose value is generally different) from the first group, making them comparable with the first group. The process is then continued. The AHP software program Expert Choice performs these functions for the user. The reason for using clusters of a few elements is to ensure greater stability of the priorities in face of inconsistent judgments. Comparing more than two elements allows for redundancy and hence also for greater validity of real-world information. The AHP often uses seven elements and puts them in clusters if

there are more. (Elaborate mathematical derivations are given in the AHP to show that the number of elements compared should not be too large in order to obtain priorities with admissible consistency.)

Problems with Analytic Decision Making

At this point you may wonder why we have three different modes for establishing priorities, the absolute measurement mode and the distributive and ideal modes of relative measurement. Isn't one enough? Let me explain why we need more than one mode.

A major reason for having more than one mode is concerned with this question. What happens to the synthesized ranks of alternatives when new ones are added or old ones deleted? With consistent judgments, the original relative rank order cannot change under any single criterion, but it can under several criteria.

Assume that an individual has expressed preference among a set of alternatives, and that as a result, he or she has developed a ranking for them. Can and should that individual's preferences and the resulting rank order of the alternatives be affected if alternatives are added to the set or deleted from it and if no criteria are added or deleted, which would affect the weights of the old criteria? What if the added alternatives are copies or near copies of one or of several of the original alternatives and their number is large? Rank reversal is an unpleasant property if it is caused by the addition of truly irrelevant alternatives. However, the addition of alternatives may just reflect human nature: the straw that broke the camel's back was considered irrelevant along with all those that went be-

fore it. Mathematically, the number and quality of newly added alternatives are known to affect preference among the original alternatives. Most people, unaided by theory and computation, make each decision separately, and they are not very concerned with rank reversal unless they are forced for some reason to refer to their earlier conclusions. I think it is essential to understand and deal with this phenomenon.

An Example of Rank Reversal

Two products A and B are evaluated according to two equally important attributes P and Q as in the following matrices:

P	A	B	Priorities
A	1	5	0.83
B	1/5	1	0.17

Q	A	B	Priorities
A	1	1/3	0.25
B	3	1	0.75

We obtain the following priorities: $W_A = 0.542$, $W_B = 0.458$, and A is preferred to B.

A third product C is then introduced and compared with A and B as follows:

P	A	B	C	Priorities
A	1	5	1	0.455
B	1/5	1	1/5	0.090
C	1	5	1	0.455

Q	A	B	C	Priorities
A	1	1/3	2	0.222
B	3	1	6	0.666
C	1/2	1/6	1	0.111

Synthesis yields W_A = 0.338, W_B = 0.379, and W_C = 0.283. Here B is preferred to A and there is rank reversal.

For a decision theory to have a lasting value, it must consider how people make decisions naturally and assist them in organizing their thinking to improve their decisions in that natural direction. Its assumptions should be tied to evolution and not to present day determinism. This is the fundamental concept on which the AHP is based. It was developed as a result of a decade of unsuccessful attempts to use normative theories, with the assistance of some of the world's best minds, to deal with negotiation and trade-off in the strategic political and diplomatic arena at the Arms Control and Disarmament Agency in the Department of State. In the early 1970s, I asked the question, how do ordinary people process information in their minds in attempting to make a decision and how do they express the strength of their judgments? The answer to this question led me to consider hierarchies and networks, paired comparisons, ratio scales, homogeneity and consistency, priorities, ranking, and the AHP.

Resolution of the Rank Preservation Issue

Early developers of utility theory axiomatically ruled that introducing alternatives, particularly "irrelevant" ones, should not cause rank reversal [Luce and Raiffa 1957]. A theory that rates alternatives one at a time, as in the absolute measurement salary-raise example given above, assumes the existence of past standards established by experts for every decision problem and would thus assume that every decision can be made by rating each alternative by itself without regard to any other alternative and would inexorably preserve rank. But if past standards are inapplicable to new problems and if experts are not sufficiently familiar with the domain of a decision to establish standards and the environment changes rapidly, an insistence on making decisions based on standards will only force the organization to shift its efforts from solving the problem to updating its standards. For example, practitioners have improvised many techniques to relate standards defined by utility functions in the context of a specific decision problem. Connecting theory to practice is important but often difficult. We need to distinguish between fixing the axioms of a decision theory to be followed strictly in all situations and learning and revising in the process of making a decision. The rank preservation axioms of utility theory and the AHP parallel the axioms of the classical frequentist method of statistics and Bayesian theory. Bayesian theory violates the axioms of statistics in updating prediction by including information from a previous outcome, a process known as learning. When we integrate learning with decision making, we question some of the static basic axioms of utility theory.

The AHP avoids this kind of formulation and deals directly with paired comparisons of the priority of importance, preference, or likelihood (probability) of pairs of elements in terms of a common attribute or criterion represented in the decision hierarchy. We believe that this is the natural (but refined) method that people followed in making decisions long before the development of utility functions and before the AHP was formally developed.

The major objection raised against the AHP by practitioners of utility theory has been this issue of rank reversal. The issues of rank reversal and preference reversal have been much debated in the literature as a problem of utility theory [Grether and Plott 1979; Hershey and Schoemaker 1980; Pommerehne, Schneider, and Zweifel 1982; Saaty 1994, Chapter 5; Tversky and Simonson 1993; Tversky, Slovic, and Kahneman 1990].

Regularity is a condition of choice theory that has to do with rank preservation. R. Corbin and A. Marley [1974] provide a utility theory example of rank reversal. It "concerns a lady in a small town, who wishes to buy a hat. She enters the only hat store in town, and finds two hats, A and B, that she likes equally well, and so might be considered equally likely to buy. However, now suppose that the sales clerk discovers a third hat, C, identical to B. Then the lady may well choose hat A for sure (rather than risk the possibility of seeing someone wearing a hat just like hers), a result that contradicts regularity." Utility theory has no clear analytical answer to this paradox nor to famous examples having to do with phantom alternatives and with decoy alternatives that arise in the field of marketing [Saaty 1994].

Because of such examples, it is clear that one cannot simply use one procedure for every decision problem because that procedure would either preserve or not preserve rank. Nor can one introduce new criteria that indicate the dependence of the alternatives on information from each new alternative that is added. In the AHP, this issue has been resolved by adding the ideal mode to the normalization mode in relative measurement. The ideal mode prevents an alternative that is rated low or "irrelevant" on all the criteria from affecting the rank of higher rated alternatives.

In the AHP, we have one way to allow rank to change, (1) below, and two ways

---

The essence of the AHP is the use of ratio scales in elaborate structures to assess complex problems.

---

to preserve rank, (2) and (3) below.
(1) We can allow rank to reverse by using the distributive mode of the relative measurement approach of the AHP.
(2) We can preserve rank in the case of irrelevant alternatives by using the ideal mode of the AHP relative measurement approach.
(3) We can preserve rank absolutely by using the absolute measurement mode of the AHP.

As a recap, in relative measurement, we use normalization by dividing by the sum of the priorities of the alternatives to define the distributive mode. In this mode, we distribute the unit value assigned to the goal of a decision proportionately among the alternatives through normalization. When we add a new alternative, it takes its share of the unit from the previously existing alternatives. This mode allows for rank reversal because dependence exists among the alternatives, which is attributable to the number of alternatives and to their measurements values and which is accounted for through normalization. For example, multiple copies of an alternative

can affect preference for that alternative in some decisions. We need to account for such dependence in allocating resources, in voting and in distributing resources among the alternatives.

In the ideal mode, we would simply compare a new alternative with the ideal (with the weight of one), and it would fall below or above the ideal and could itself become the ideal. As a result, an alternative that falls far below the ideal on every criterion cannot affect the rank of the best chosen alternative. Using absolute measurement, we rate alternatives one at a time with respect to an ideal intensity on each criterion, and this process cannot give rise to rank reversal.

I conducted an experiment involving 64,000 hierarchies with priorities assigned randomly to criteria and to alternatives to test the number of times the best choice obtained by the distributive and ideal modes coincided with each other. It turns out that the two methods yield the same top alternative 92 percent of the time. I obtained similar results for the top two alternatives [Saaty and Vargas 1993a].

Decision Making in Complex Environments

The AHP makes group decision making possible by aggregating judgments in a way that satisfies the reciprocal relation in comparing two elements. It then takes the geometric mean of the judgments. When the group consists of experts, each works out his or her own hierarchy, and the AHP combines the outcomes by the geometric mean. If the experts are ranked according to their expertise in a separate hierarchy, we can raise their individual evaluations to the power of their importance or expertise

priorities before taking the geometric mean. We have also used special questionnaires to gather data in the AHP.

Practitioners have developed multicriteria decision approaches largely around techniques for generating scales for alternatives. But I believe that making decisions in real life situations depends on the depth and sophistication of the structures decision makers use to represent a decision or prediction problem rather than simply on manipulations—although they are also important. It seems to me that decision making and prediction must go hand in hand if a decision is to survive the test of the forces it may encounter [Saaty and Vargas 1991]. If one understands the lasting value of a best decision, one will want to consider feedback structures with possible dependencies among all the elements. These would require iterations with feedback to determine the best outcome and the most likely to survive. I believe that ratio scales are mathematically compelling for this process. The AHP is increasingly used for decisions with interdependencies (the hierarchic examples I have described are simple special cases of such decisions). I describe applications of feedback in Chapter 8 of Saaty [1994] and in a book I am currently writing on applications of feedback. I and my colleague Luis Vargas used the supermatrix feedback approach of the AHP in October 1992 to show that the well-known Bayes theorem used in decision making follows from feedback in the AHP. Furthermore, we have since shown through examples that some decisions with interdependence can be treated by the AHP but not Bayes theorem [Saaty and Vargas 1993b].

The essence of the AHP is the use of ratio scales in elaborate structures to assess complex problems. Ratio scales are the fundamental tool of the mind that people use to understand magnitudes. The AHP well fits the words of Thomas Paine in his Common Sense, "The more simple anything is, the less liable it is to be disordered and the easier repaired when disordered."

In August 1993, Sarah Becker compiled a list of what are now more than 1,000 papers, books, reports, and dissertations written on the subject of AHP, an early version of which is included as a bibliography in my 1994 book [Saaty 1994].

The Benefits of Analytic Decision Making

Many excellent decision makers do not rely on a theory to make their decisions. Are their good decisions accidental, or are there implicit logical principles that guide the mind in the process of making a decision, and are these principles complete and consistent? I believe that there are such principles, and that in thoughtful people, they work as formalized and described in the analytic hierarchy process. Still academics differ about how people should and should not make decisions. Experiments with people have shown that what people do differs from the theoretical and normative considerations the experts consider important. This may lead one to believe that analytical decision making is of little value. But our experience and that of many others indicate the opposite.

Analytic decision making is of tremendous value, but it must be simple and accessible to the lay user, and must have scientific justification of the highest order. Here are a few ideas about the benefits of the descriptive analytical approach. First is the morphological way of thoroughly modeling the decision, inducing people to make explicit their tacit knowledge. This leads people to organize and harmonize their different feelings and understanding. An agreed upon structure provides ground for a complete multisided debate. Second, particularly in the framework of hierarchies and feedback systems, the process permits decision makers to use judgments and observations to surmise relations and strengths of relations in the flow of interacting forces moving from the general to the particular and to make predictions of most likely outcomes. Third, people are able to incorporate and trade off values and influences with greater accuracy of understanding than they can using language alone. Fourth, people are able to include judgments that result from intuition and emotion as well as those that result from logic. Reasoning takes a long time to learn, and it is not a skill common to all people. By representing the strength of judgments numerically and agreeing on a value, decision-making groups do not need to participate in prolonged argument. Finally, a formal approach allows people to make gradual and more thorough revisions and to combine the conclusions of different people studying the same problem in different places [Saaty and Alexander 1989]. One can also use such an approach to piece together partial analyses of the components of a bigger problem, or to decompose a larger problem into its constituent parts. This is not an exhaustive list of the uses of the AHP. However, to deal with complexity we need rationality, and that is best manifested in the analytical approach.

APPENDIX

The AHP has four axioms: (1) reciprocal judgments, (2) homogeneous elements, (3) hierarchic or feedback dependent structure, and (4) rank order expectations [Saaty 1986].

Assume that one is given n stones, A_1, ..., A_n, with known weights $w_1, \ldots, w_n$, respectively, and suppose that a matrix of pairwise ratios is formed whose rows give the ratios of the weights of each stone with respect to all others. Thus one has the equation:

$$Aw = \begin{array}{c} A_1 \\ \vdots \\ A_n \end{array} \begin{bmatrix} \dfrac{w_1}{w_1} & \cdots & \dfrac{w_1}{w_n} \\ \vdots & & \vdots \\ \dfrac{w_n}{w_1} & \cdots & \dfrac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$= n \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = nw.$$

where A has been multiplied on the right by the vector of weights w. The result of this multiplication is nw. Thus, to recover the scale from the matrix of ratios, one must solve the problem $Aw = nw$ or $(A - nI)w = 0$. This is a system of homogeneous linear equations. It has a nontrivial solution if and only if the determinant of $A - nI$ vanishes, that is, n is an eigenvalue of A. Now A has unit rank since every row is a constant multiple of the first row. Thus all its eigenvalues except one are zero. The sum of the eigenvalues of a matrix is equal to its trace, the sum of its diagonal elements, and in this case the trace of A is equal to n. Thus n is an eigenvalue of A, and one has a nontrivial solution. The solution consists of positive entries and is unique to within a multiplicative constant. To make w unique, one can normalize its entries by dividing by their sum. Thus,

given the comparison matrix, one can recover the scale. In this case, the solution is any column of A normalized. Notice that in A the reciprocal property $a_{ji} = 1/a_{ij}$ holds; thus, also $a_{ii} = 1$. Another property of A is that it is consistent: its entries satisfy the condition $a_{jk} = a_{ik}/a_{ij}$. Thus the entire matrix can be constructed from a set of n elements which form a chain across the rows and columns.

In the general case, the precise value of w_i/w_j cannot be given, but instead only an estimate of it as a judgment. For the moment, consider an estimate of these values by an expert who is assumed to make small perturbations of the coefficients. This implies small perturbations of the eigenvalues. The problem now becomes $A'w' = \lambda_{max}w'$ where λ_{max} is the largest eigenvalue of A'. To simplify the notation, we shall continue to write $Aw = \lambda_{max}w$, where A is the matrix of pairwise comparisons. The problem now is how good is the estimate of w. Notice that if w is obtained by solving this problem, the matrix whose entries are w_i/w_j is a consistent matrix. It is a consistent estimate of the matrix A. A itself need not be consistent. In fact, the entries of A need not even be transitive; that is, A_1 may be preferred to A_2 and A_2 to A_3 but A_3 may be preferred to A_1. What we would like is a measure of the error due to inconsistency. It turns out that A is consistent if and only if $\lambda_{max} = n$ and that we always have $\lambda_{max} \geq n$.

Since small changes in a_{ij} imply a small change in λ_{max}, the deviation of the latter from n is a deviation from consistency and can be represented by $(\lambda_{max} - n)/(n - 1)$, which is called the *consistency index* (C.I.). When the consistency has been calculated, the result is compared with those of the same index of a randomly generated reciprocal matrix from the scale 1 to 9, with reciprocals forced. This index is called the *random index* (R.I.). Table 11 gives the order of the matrix (first row) and the average R.I. (second row).

n	1	2	3	4	5	6	7	8	9	10
Random Consistency Index (R.I.)	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Table 11: The order of the matrix (first row) and the average R. I. (second row).

The ratio of C.I. to the average R.I. for the same order matrix is called the *consistency ratio* (C.R.). A consistency ratio of 0.10 or less is positive evidence for informed judgment.

The relations $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$ are preserved in these matrices to improve consistency. The reason for this is that if stone #1 is estimated to be k times heavier than stone #2, one should require that stone #2 be estimated to be $1/k$ times the weight of the first. If the consistency ratio is significantly small, the estimates are accepted; otherwise, an attempt is made to improve consistency by obtaining additional information. What contributes to the consistency of a judgment are (1) the homogeneity of the elements in a group, that is, not comparing a grain of sand with a mountain; (2) the sparseness of elements in the group, because an individual cannot hold in mind simultaneously the relations of many more than a few objects; and (3) the knowledge and care of the decision maker about the problem under study.

Figure 4 shows five areas to which we can apply the paired comparison process in a matrix and use the 1–9 scale to test the validity of the procedure. We can approximate the priorities in the matrix by assuming that it is consistent. We normalize each column and then take the average of the corresponding entries in the columns.

The actual relative values of these areas are $A = 0.47$, $B = 0.05$, $C = 0.24$, $D = 0.14$, and $E = 0.09$ with which the answer may be compared. By comparing more than two alternatives in a decision problem, one is able to obtain better values for the derived scale because of redundancy in the comparisons, which helps improve the overall accuracy of the judgments.

Figure 4: Five figures drawn with appropriate size of area. The object is to compare them in pairs to reproduce their relative weights.

References

Corbin, R. and Marley, A. A. J. 1974, "Random utility models with equality: An apparent, but not actual, generalization of random utility models," *Journal of Mathematical Psychology,* Vol. 11, No. 3, pp. 274–293.

Grether, D. M. and Plott, C. R. 1979, "Economic theory of choice and the preference reversal phenomenon," *The American Economic Review,* Vol. 69, No. 4, pp. 623–638.

Hershey, J. C. and Schoemaker, P. J. H. 1980, "Prospect theory's reflection hypothesis: A critical examination," *Organization of Behavioral Human Performances,* Vol. 25, No. 3, pp. 395–418.

Luce, R. D. and Raiffa, H. 1957, *Games and Decisions,* John Wiley and Sons, New York.

Pommerehne, W. W.; Schneider, F.; and Zweifel, P. 1982, "Economic theory of choice and the preference reversal phenomenon: A reexamination," *The American Economic Re-*
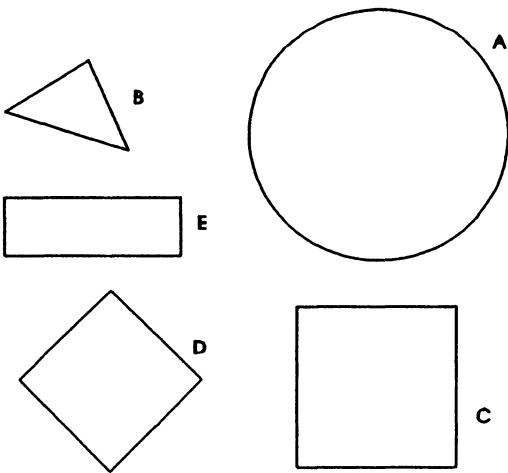
view, Vol. 72, No. 3, pp. 569–574.

Saaty, T. L. 1977, "A scaling method for priorities in hierarchical structures," *Journal of Mathematical Psychology*, Vol. 15, No. 3, pp. 234–281.

Saaty, T. L. 1982, *Decision Making for Leaders*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, Pennsylvania.

Saaty, T. L. 1986, "Axiomatic foundation of the analytic hierarchy process," *Management Science*, Vol. 32, No. 7, pp. 841–855.

Saaty, T. L. 1994, *Fundamentals of the Analytic Hierarchy Process*, RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, Pennsylvania.

Saaty, T. L. and Alexander, J. 1989, *Conflict Resolution: The Analytic Hierarchy Process*, Praeger, New York.

Saaty, T. L. and Vargas, L. G. 1991, *Prediction, Projection and Forecasting*, Kluwer Academic Publishers, Boston, Massachusetts.

Saaty, T. L. and Vargas, L. G. 1993a, "Experiments on rank preservation and reversal in relative measurement," *Mathematical and Computer Modelling*, Vol. 17, No. 4/5, pp. 13–18.

Saaty, T. L. and Vargas, L. G. 1993b, "Diagnosis with dependent symptoms: Bayes theorem derived from the analytic hierarchy process," working paper, University of Pittsburgh, Pittsburgh, Pennsylvania.

Tversky, A. and Simonson, I. 1993, "Context-dependent preferences," *Management Science*, Vol. 39, No. 10, pp. 1179–1189.

Tversky, A.; Slovic, P.; and Kahneman, D. 1990, "The causes of preference reversal," *The American Economic Review*, Vol. 80, No. 1, pp. 204–215.